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11204. *Proposed by Christopher Hillar, Texas A& M University, College Station, TX.* For integers m and j with $m \geq j \geq 0$, and square matrices X and Y of the same size, let $H_{m,j}(X, Y)$ denote the sum of all products of the form $A_1 \cdots A_m$ such that each A_i is either X or Y, and is Y in exactly j cases (by convention, we set $H_{0,0}$ to be the identity matrix). Let $\mathrm{tr}(A)$ denote the trace of A. Prove that for all (m, j) with $m > j \geq 0$ there exists a constant $c(m, j)$ such that for all complex square matrices X and Y of the same size,

$$\mathrm{tr}[H_{m,j}(X, Y)] = c(m, j)\mathrm{tr}[XH_{m-1,j}].$$