JUNE-JULY 2006 AMERICAN MATHEMATICAL MONTHLY PROBLEM

11231. Proposed by Christopher Hillar, Texas A& M University, College Station, TX. Find a non-Abelian group G with the following property: for each n, every word W on the alphabet of n+1 letters A_1, \ldots, A_n , and X, all lists a_1, \ldots, a_n of elements of G, and every b in G there exists a unique x in G such that $W(a_1, \ldots, a_n, x) = b$. (Thus, in particular, $ax^2ax = b$ must have a unique solution x.)