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ABSTRACT. [11288] For a set $S \subseteq \{1, \dots, m\}$ and a positive integer d , let h_S^d be the sum of all monomials of degree d in the indeterminates $\{x_i : i \in S\}$. Also, let $o(S)$ denote the number of odd integers in S . Fix a positive integer n and let $U = \{1, \dots, 2n\}$. Prove that

$$(0.1) \quad h_U^d \prod_{i=1}^n (x_{2i-1} - x_{2i}) = \sum_S (-1)^{o(S)} h_{U \setminus S}^{d+n},$$

where this sum is over all $S \subseteq U$, $|S| = n$, with the property that $\{2i-1, 2i\} \not\subseteq S$ for every $i = 1, \dots, n$.

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