MATHEMATICAL MONTHLY PROBLEM PROPOSAL

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Christopher Hillar, Texas A& M University, College Station, TX, and Lionel Levine, University of California Berkeley, Berkeley, CA. Let n be an integer greater than 1 and let $S = \{2, \ldots, n\}$. For each nonempty subset A of S, let $\pi(A) = \prod_{j \in A} j$.

Prove that when k is a positive integer and $k < n$,

$$\prod_{i=k}^{n} \operatorname{lcm}(\{1, \ldots, \lfloor n/i \rfloor\}) = \gcd\left(\{\pi(A) \colon\ |A| = n - k\}\right).$$

(In particular, setting $k = 1$ yields $\prod_{i=1}^{n} \operatorname{lcm}(\{1, \ldots, \lfloor n/i \rfloor\}) = n!$.)

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